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Optimizing energy storage devices using Ragone plots

Thomas Christen*, Christian Ohler

ABB Corporate Research Ltd., CH-5405, Baden-Dättwil, Switzerland

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Abstract

This paper describes how to optimize energy storage devices (ESDs) by maximizing their net present value (NPV). This requires both technical and economic information. The relevant technical information is specified in concise form by the energy–power relation (Ragone-plot) of the ESD and its lifetime. The economic information is given in terms of operation costs (energy costs), investment costs, and the economic benefit created by operating the ESD. The NPV is expressed as a function of variables such as the size of the ESD. An appropriate choice of these variables maximizes the NPV. If the benefit is given in terms of the energy supplied by the ESD and the lifetime is operation independent, the optimization reduces to a minimization of the total lifecycle costs. In this case a knowledge of the economic benefit, which is often the quantity that is most difficult to model, is not required. In more general cases, e.g. if the lifetime depends on optimization variables, a quantification of the benefit is necessary in order to maximize the NPV. We illustrate the approach with various stationary and mobile applications, and for batteries and capacitors.

Keywords: Ragone-plot; Batteries; Supercapacitors; Economic optimization; Net present value

1. Introduction

To optimize the design of an energy storage device (ESD) means different things to different people. In this article, it refers to a maximization of its net present value (NPV) [1]. The NPV is the value of all positive cashflows (revenues) less the value of all negative cashflows (costs) both discounted to a single date to make them commensurable. Mathematically, the NPV is a weighted sum of functions containing technical information (efficiency, power, mass, etc.), where the weights reflect economic importance in monetary terms (costs or revenues). The optimization task requires, thus, business knowledge, and engineering know-how in an interdisciplinary manner. Depending on the particular energy storage design problem to be solved it can be anything between straightforward and akward to translate technical duties into monetary terms. A straightforward example is trading of electrical energy, but what would be, e.g. the dollar value of having a generously sized starter battery in a car? The concept of NPV is, however, general enough to allow the description of a wide variety of energy storage problems.

In this paper, we use a particular simplifying and meaningful description of ESDs for optimization: energy–power relations, the so called Ragone plots. Ragone plots are curves which relate the power (density) of an ESD to the available energy (density). Due to internal losses, the energy available for use is in general less than the energy stored in the ESD. The question arises: what is the optimum operation point along the Ragone curve for a given application? Is it the maximum possible power point (“fast discharge”), or the maximum possible energy point (“slow discharge”), or what compromise should be made between the two? This question can only be answered within a combined technical and economical description. Ragone plots have so far been mainly used for a rough comparison of energy storage technologies across orders of magnitude in either power or energy capability. However, with sufficient care in the definition and sufficient accuracy in the measurement of Ragone plots, they may serve as a realistic conceptual tool for the actual design of energy storage units. On the other hand, NPV and Ragone plots are both abstract concepts. In some long established energy storage problems, they sometimes likely produce only expected or even trivial answers. In less known engineering terrain, however, where experience and intuition is not yet mature enough, the concepts discussed in the present paper can serve indeed as a sound starting point for a deeper analysis.

The paper is organized as follows. The next section provides the nomenclature and the general expression for the NPV of an ESD. Section 3 reviews some basics of Ragone plots. Section 4 explains how to solve for the optimum size of the ESD. The remaining part of the paper is devoted to

* Corresponding author. Tel.: +41-(0)58-586-82-07; fax: +41-(0)58-588-00-09.
E-mail address: thomas.christen@ch.abb.com (T. Christen).

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illustrative examples going from easier examples to more complex ones. Throughout this paper, we consider two types of energy–power relations, which describe a battery and a capacitor. The most simple application (Section 5) is an ESD for a stationary application with constant power demand. After providing some numerical examples in Section 6, the effect of a simple time-dependent power demand is discussed (Section 7). Then, in Section 8, we investigate a mobile application, where the power is partly used to carry the ESD itself. For these three examples the benefit is given in terms of the energy supplied by the ESD, and a constant lifetime is assumed. As a consequence, the NPV maximization is equivalent to a minimization of the total lifecycle costs. In Section 9, we generalize our approach to an example where the lifetime does depend on the operation point. This case requires information on the economic benefit of operating the ESD if the NPV is to be maximized. As an example, we compare in Section 9, the NPV with the return on investment (ROI). In Section 10, we finally discuss a different direction of generalization, namely a dependence of the benefit on the operation point. This is a rather general case that will be illustrated for an ESD for uninteruptible power supply (UPS).

2. The net present value of an energy storage device

In this section, we derive an expression for the NPV, the sum of all discounted cashflows. Because the purpose is to optimize the design by maximizing the NPV, the considerations will be restricted to those parts that depend on the design variables under consideration. The absolute NPV is irrelevant in this context because constant, i.e. design independent value components cannot have an impact on design decisions (except the decision whether to have an ESD at all). Those constant components vanish on differentiation. The NPV consists of the present value of all benefits B_{LC} (revenues) minus the present value of all costs C_{LC} during the lifetime of the ESD:

$$\mathrm{NPV} = B_{\mathrm{LC}} - C_{\mathrm{LC}}. \tag{1}$$

The lifecycle costs, C_{LC}, are usually arranged into two main groups, initial investment costs, C_{inv}, at time $t = 0$, and operation costs, C_{op}, that occur during the system lifetime τ:

$$C_{\mathrm{LC}} = C_{\mathrm{inv}} + C_{\mathrm{op}}. \tag{2}$$

The unit of the costs will be $[C] = \$$.

We restrict our analysis to ESDs which are scalable in size. The investment costs therefore contain a term dependent on the system size. We furthermore assume this dependency to be linear such that the ESD consists of a number N of units with costs c_N \$ per unit. Examples for units may be volume ($[N] = \mathrm{m}^3$, $[c_N] = \$/\mathrm{m}^3$), mass ($[N] = \mathrm{kg}$, $[c_N] = \$/\mathrm{kg}$), or number of cells in a battery or capacitor bank ($[N] = 1$, $[c_N] = \$$ per cell), etc. The investment costs are then

$$C_{\mathrm{inv}} = C_{\mathrm{inv}}^0 + c_N N \tag{3}$$

where C_{inv}^0 is the size independent part of the investment costs. The NPV analysis can also be conducted for the case of a non-linear relationship between investment costs and size and also for the dependence of investment costs on design parameters other than just the size. Investment costs should also include the present value of the costs for dismantle and disposal at the end of lifetime.

Again, we make a restriction that can be easily lifted and assume that operation costs, $C_{\mathrm{op},n}$ incur each year ($0 < n \le \tau$; $n, \tau = 1, 2, \ldots$) at the same rate during the product lifetime τ. In order to add the yearly contributions $C_{\mathrm{op},n}$, one has to calculate their present value at the time of investment. Denoting by r the cost of capital in percent per year, one has

$$C_{\mathrm{op}} = \sum_{n=1}^{\tau} \frac{C_{\mathrm{op},n}}{(1+r)^{n-1}}. \tag{4}$$

In the most general case, both, operation costs and operation benefits can depend on a variety of parameters. For the sake of clarity, we make some further simplifying restrictions. We assume that the operation costs can be separated in a part associated with the cost of the energy stored in the ESD and a part, C_{op}^0, containing the rest which is independent of the amount of energy stored (maintenance costs, etc.). If the fuel costs and the load characteristics are the same for each year, and if aging (i.e. time dependence) of the efficiency can be neglected, one can write the operation costs in the form:

$$C_{\mathrm{op}} = C_{\mathrm{op}}^0 + c_{\mathrm{e}} d \int_{1\,\mathrm{year}} \frac{P_{\mathrm{req}}}{\eta}\,\mathrm{d}t, \tag{5}$$

where $P_{\mathrm{req}}(t)$ is the power demand, i.e. the power ($[P_{\mathrm{req}}] = W$) delivered by the ESD to the load at time t, c_{e} is the cost of energy ($[c_{\mathrm{e}}] = \$/\mathrm{J}$), and η is the total (“round-trip”) energy efficiency, i.e. the amount of energy that is delivered from the ESD divided by the amount of energy used to charge up the ESD. This means that the total energy efficiency includes the charging efficiency. While η depends on the required power and the size of the ESD, the frequency dependence of the efficiency is neglected. The case where the time variation of $P_{\mathrm{req}}(t)$ is so fast that frequency dependence becomes important will be discussed elsewhere. Furthermore, for constant r, d is

$$d = \sum_{n=1}^{\tau} \frac{1}{(1+r)^{n-1}} = \frac{v^\tau - 1}{v - 1}, \tag{6}$$

with $v = 1/(1+r)$. Note that $d = d(\tau)$ is a function of the lifetime and may thus depend on the operation conditions. In general, this fact must be taken into account in an optimization. But the specific dependence is rather difficult to model, since it depends in general on the full load-profile history. Only in the case of a constant power demand, τ can become a simple function of the power P. Summarizing, the total lifecycle costs are

$$C_{\mathrm{LC}} = C_{\mathrm{LC}}^0 + c_N N + c_{\mathrm{e}} d \int_{1\,\mathrm{year}} \frac{P_{\mathrm{req}}}{\eta}\,\mathrm{d}t, \tag{7}$$

where C_{LC}^0 contains all contributions, which are not contained in the size dependent investment costs or in the energy related operation costs. For simplicity, we will omit C_{LC}^0, it does not add conceptual difficulties.

We must also find an expression for the total monetary benefits B_{LC} due to providing energy from the ESD. This expression can look very different depending on circumstances, it can pose modelling challenges being, e.g. a volatile market price or the benefit of riding through a power outage. For the sake of clarity, we use a simple description with fixed price per output energy, say b_e ($[b_e] = \$/\mathrm{J}$). Note that the pure energy margin (i.e. excluded investment costs) is not $b_e - c_e$ but $b_e - c_e/\eta$, since there is always an energy loss ($\eta < 1$). With assumptions analogous to those used for the derivation of the operation costs, we obtain

$$B_{LC} = b_e d \int_{1\,\mathrm{year}} P_{req}\,\mathrm{d}t. \tag{8}$$

The remaining task of this paper is mainly to find the maximum of Eq. (1) with Eqs. (7) and (8). Before we conclude this section, we mention that the only dependence of B_{LC} on the optimization variables in our restricted scenario enters via the lifetime through $d(\tau)$. Hence, if the dependence of the lifetime on the operation conditions is negligible (in the region of interest), the maximization of the NPV reduces to the minimization of Eq. (7). In other words, NPV analysis contains the lifecycle cost analysis as a special case. This is particularly advantageous, since it eliminates the difficulty of modelling B_{LC}. In the last example, we consider the general case of an operation dependent B_{LC}, which reflects another benefit structure of the ESD for a user than the revenues from energy supply.

3. Ragone plots

The physical information on an ESD comes in a variety of disguises. It is connected to quantities like capacitance or capacity, internal resistance, efficiencies, power density, lifetime, etc. This makes a comparison between energy storage technologies often cumbersome. Energy–power relations, Ragone plots, have a two-fold advantage as a concept for ESD optimization: they are rigorously defined for any kind of ESD [2] and they readily display the two parameters with cost impact. The energy efficiency affects the energy costs, while the investment costs for a specific application depend on both the power and the energy density. The information on efficiency, energy, and power density is contained in the Ragone plot, which is briefly reviewed in this section.

Energy storage technologies are characterized by a typical power scale and a typical energy scale, which together determine their natural field of applications. For example, batteries have high energy densities and are used for long-time applications in the range of hours, while electric double layer supercapacitors have high power densities and are used for short-time applications down to fractions of a second. It is convenient to compare different technologies in the energy–power plane. A specific ESD does not correspond to a single point in this plane, but is represented by a curve, which displays the energy E available to a load as a function of the power P, the rate with which the energy is supplied to the load. This energy–power relation is known as Ragone-plot [2–6]. The exact definition of an energy–power relation may differ among different publications [7]. Usually, these relations are plotted on a log–log scale with units specific energy ($[E] = \mathrm{J/kg}$) versus specific power ($[P] = \mathrm{W/kg}$), or energy density ($[E] = \mathrm{J/m^3}$) versus power density ($[P] = \mathrm{W/m^3}$).

The energy–power relation, $E(P)$, can be used to express the energy costs in Eq. (7). Here, we define E and P with respect to the above introduced "unit" of N (e.g. specific energy, energy density, energy per unit cell, etc.), such that the power P delivered by a single unit in a ESD of size N is P_{req}/N. If E_0 is the initial energy (per unit) of the charged ESD, $E(P)/E_0$ is the discharge energy efficiency. Throughout this paper, we make the simplification that the efficiency (in contrast to the lifetime) does not depend on the discharge depth. By introducing a charging efficiency η_c, the total energy efficiency can finally be expressed by the energy–power relation:

$$\eta = \frac{E(P)}{E_0}\eta_c = \frac{E(P_{req}/N)}{E_0}\eta_c = \eta(P_{req}/N). \tag{9}$$

Later, we will assume a constant charging efficiency η_c without explicit power dependence, except for a single example with charge–discharge symmetry, where $\eta_c = E(P)/E_0$, i.e. charging and discharging is done at the same power.

We will always make use of two examples of ESD for illustration of the theory, namely the (rechargeable) battery and the (super)capacitor. Both are characterized by their energy capacity, E_0, per unit and by their maximum power P_{max} per unit. The internal loss will be described by an ohmic resistance R, such that $P_{max} = U_0^2/4R$, where U_0 is the cell voltage of the charged-up state. The energy capacities are $E_0 = Q_0U_0$ for the battery with charge capacity Q_0 per cell, and $CU_0^2/2$ for the capacitor with capacitance C per cell. The Ragone plots of these examples are derived and discussed in Ref. [2]. It is convenient to represent them in dimensionless units, $p = P/P_{max}$ and $e(p) = E(pP_{max})/E_0$, with $0 \le p, e \le 1$. The efficiency as a function of p is given by $\eta = \eta_c e(p)$. The energy–power relation for an ideal battery is [2] given as

$$e(p) = \frac{1}{2}\frac{p}{(1-\sqrt{1-p})}, \tag{10}$$

and a convenient approximation of the Ragone-plot for a capacitor-like ESD is [7] (we do not consider the exact, more complicated expression given in [2])

$$e(p) = 1 - p. \tag{11}$$

The two functions (10) and (11) are shown in Fig. 1. The main difference between them is that the available energy

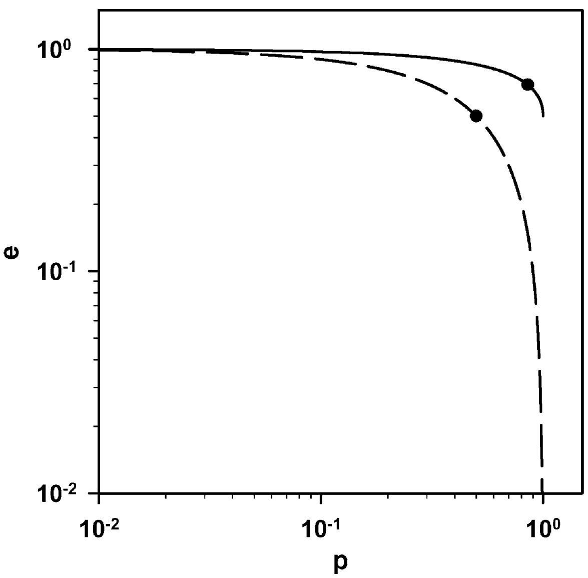

Fig. 1. Normalized Ragone plots for a battery (solid curve) and a capacitor (dashed curve). The energy e is normalized to the energy capacity (e has then the meaning of a discharge efficiency), and the power p to the maximum power. The dots indicate the optimum operation point for $K = 1$ (cf. Section 5).

vanishes at maximum power for the capacitor, while the available energy remains finite for battery-like ESD; more exactly, it is one half of the full energy. Leakage [2] will be neglected. Leakage leads to a downturn of the Ragone curve at very low power. The optimum operation point of a reasonable ESD is usually far out of this leakage region. We re-emphazise that our examples are for illustration only; for realistic cases the optimization should be carried out with measured energy efficiencies or measured energy–power relations.

4. The optimization problem

In order to determine the optimum design of an ESD, we express the NPV as a function of the design variables. Consider, e.g. the size N of the ESD. Optimization means then replacement of P by P_{req}/N in the NPV and

$$\mathrm{NPV}(N) = \max! \tag{12}$$

where

$$\mathrm{NPV}(N) = b_e d \int_{1\,\mathrm{year}} P_{req}\,\mathrm{d}t - c_e d \int_{1\,\mathrm{year}} \frac{P_{req}}{\eta}\,\mathrm{d}t - c_N N - C^0_{LC}. \tag{13}$$

The first term on the right-hand side refers to the benefit, which is related to the energy supplied by the ESD. This term can be easily generalized and can be an arbitrary function of N and, if necessary, of other variables appearing in the problem. In many realistic cases, there exists a local quadratic maximum ($\mathrm{d(NPV)}/\mathrm{d}N = 0$) which turns out to be also the global maximum. In order to avoid the above mentioned problem of modelling an operation dependent lifetime τ as a function of the whole history of the ESD, we assume that it can be modelled by a reasonable function $\tau(N)$ for a given P_{req}. Then, Eq. (13) is a well-defined function of N and the determination of the maximum is straightforward. The most simple case refers to a constant lifetime τ independent of operation power. Optimization of the NPV is then equivalent to a minimization of the lifecycle costs. Combination of the Eqs. (12) and (13) gives

$$N^2 + \frac{c_e d}{c_N} \int_{1\,\mathrm{year}} \frac{P^2_{req}\eta'(P_{req}/N)}{\eta^2(P_{req}/N)}\,\mathrm{d}t = 0, \tag{14}$$

where the prime means differentiation with respect to the argument of the primed function, and where η is given by Eq. (9). One observes that besides the efficiency and the load function $P_{req}(t)$ there is only one single parameter that characterizes the optimum. In dimensionless units, this parameter has the meaning of the ratio of energy costs to investment costs. For more general cases, additional parameters may enter. In the following sections, specific examples will be investigated.

5. ESD for constant power demand

Consider a stationary application with a time independent power request P_{req} during the discharge time. The required energy per year is αP_{req} with α being the utilization time per year. During part of the remaining time of the year, the ESD is charged with a charging efficiency η_c. Using $N = P_{req}/pP_{-max}$, the NPV can be written in the form

$$\mathrm{NPV} = \frac{c_N P_{req}}{P_{max}}\left(\left(\mu - \frac{1}{e}\right)K - \frac{1}{p}\right) \tag{15}$$

where we introduced the parameters

$$K = \frac{c_e \alpha d P_{max}}{c_N \eta_c} \tag{16}$$

and

$$\mu = \frac{b_e \eta_c}{c_e}. \tag{17}$$

Note that the units of P_{req} and P_{max} are watt and watt per unit, respectively, and that the optimization problem is independent of P_{req}. The meaning of K is the ratio between total effective energy costs per unit at maximum power (more exactly: the lifetime sum of energy costs for the specific utilization factor) and the investment costs per unit. The meaning of μ is the ratio between the energy benefit (the energy sales price) and energy costs (taking into account the charging loss). Since d is constant, maximum NPV corresponds to minimum lifecycle costs which can be written as

$$\frac{1}{Kp} + \frac{1}{e(p)} = \min! \tag{18}$$

The only parameter occuring in this optimization problem is K. If $e(p)$ is a decreasing function a local minimum of Eq. (18) may be expected. Minimization of Eq. (18) leads to

$$Kp^2e' + e^2 = 0, \tag{19}$$

where the prime denotes differentiation with respect to p. From Eq. (19) one finds the relation between K and the optimum operation point p:

$$p = \tfrac{1}{2}(2 + K^{-1} - \sqrt{4K^{-1} + K^{-2}})\sqrt{4K^{-1} + K^{-2}}, \tag{20}$$

for the battery (Eq. (10)), and

$$p = \frac{1}{1 + \sqrt{K}}, \tag{21}$$

for the capacitor (Eq. (11)). The functions $p(K)$ and $e(p(K))$ are plotted in Figs. 2 and 3, respectively. For every K there is an optimum operation point along the Ragone-plot. The optimum operation point for $K = 1$ is indicated in Fig. 1 by solid dots. An ESD is only useful if the maximum NPV is positive. This sets a lower limit to the energy benefit (the energy price obtained by discharging the ESD), the minimum energy benefit b_e^{min}. The relevant relation between μ and K is defined by the equations NPV = 0, and d(NPV)/dN = 0. The solution of these equations must in general be calculated numerically. Specific examples with the assumption that Eq. (15) is the total NPV and other fixed costs are negligible are shown in Fig. 4. One observes, that for fixed investment costs per power, the minimum energy benefit b_e^{min} for batteries is lower than for supercaps. But note that usually, supercapacitors have lower c_N/P_{max}.

In order to get an impression on the typical qualitative dependence of b_e^{min} on the various parameters, we provide

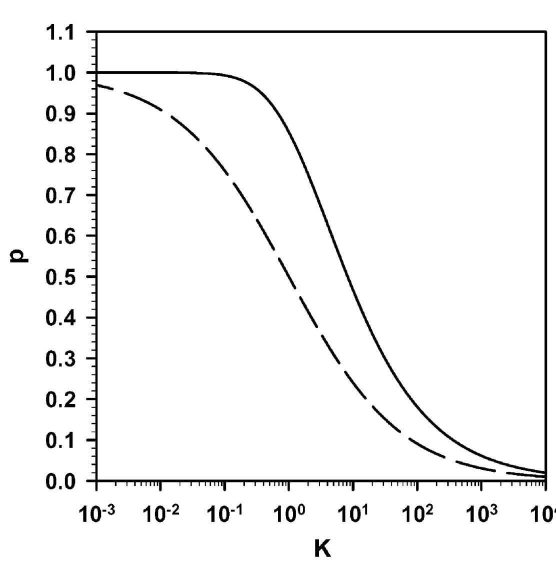

Fig. 2. Power value (p) of the optimum operation point as a function of K, for constant power request (battery: solid curve; capacitor: dashed curve).

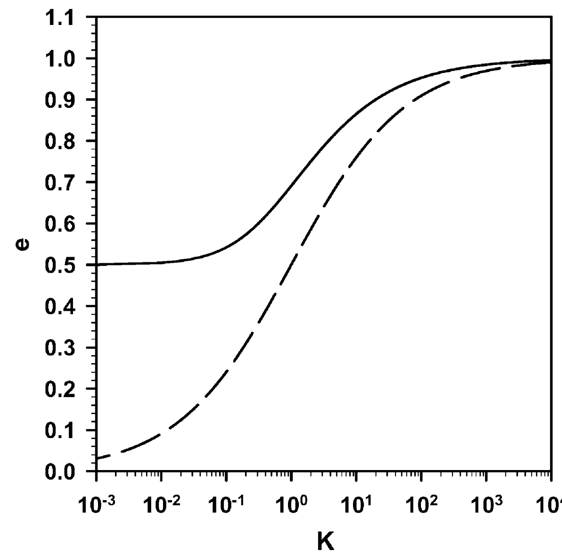

Fig. 3. Energy value (e) of the optimum operation point as a function of K, for constant power request (battery: solid curve; capacitor: dashed curve).

the result for the capacitor, which can be analytically expressed:

$$b_e^{min} = \left(\sqrt{\frac{c_N}{\alpha P_{max} d}} + \sqrt{\frac{c_e}{\eta_c}}\right)^2. \tag{22}$$

The two limit cases $K \to 0$ and $K \to \infty$ are as one expects. If investment costs strongly dominate, the present value of the revenues (per unit) have to be at least equal to the investment costs (per unit), $d\alpha P_{max} b_e^{min} = c_N$. On the other hand, if the energy costs dominate, the energy benefit must at

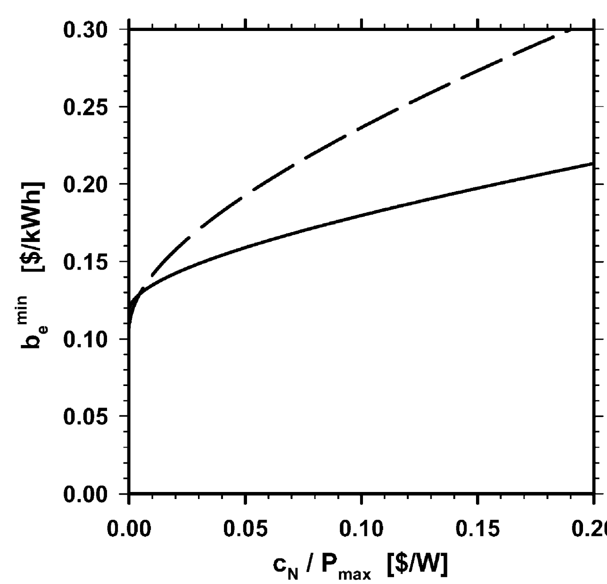

Fig. 4. Minimum total energy price as a function of the investment costs per Watt for energy costs $c_e = 0.1$ \$/kWh, discount rate $r = 0.1$, and utilizaton time $\alpha = 2000$ h per year. Solid curve: lead-acid battery ($\eta_c = 0.85$, $\tau = 2.5$ years). Dashed curve: capacitor ($\eta_c = 0.95$, $\tau = 2$ years).

least equal the costs of the stored energy $b_e^{\min} = c_e/\eta_c$ (taking into account the charging loss).

Up to now, the considerations were restricted to a given constant charging efficiency η_c. We briefly discuss the important case of charging–discharging symmetry, where $\eta_c = e(p)$ is determined by optimization. Using this in Eq. (15), optimization leads to

$$2\tilde{K}p^2e' + e^3 = 0, \quad (23)$$

where

$$\tilde{K} = K\eta_c = \frac{c_e \alpha d}{P_{\max}} c_N. \quad (24)$$

Eq. (23) should be compared with Eq. (19) and leads to qualitative similar results. The quantity which corresponds to K is in this case $2\tilde{K}/e$ or $2K\eta_c/e$. In other words, the operation costs have two times the weight as compared to the case where the charging efficiency is a fixed external parameter that is not influenced by the design.

6. Numerical examples

Before generalizing the previous example, we provide a few typical values of K, the optimum operation point, and the minimum energy benefit. In Table 1 various battery types are listed together with a typical supercapacitor. Note that there are many different types of lead-acid, Ni–Cd, and Ni–MeH batteries and of supercapacitors, and the numbers presented are only for illustration. We assume for all examples $\alpha = 2000$ h utilization time per year, energy costs $c_e = 0.1$ \$/kWh, and an interest rate $r = 10\%$. We consider Ni–MeH and Ni–Cd batteries which are similar and differ mainly in the lifetime and the efficiency. It turns out that the lead-acid battery corresponds to the highest K value among the batteries, implying that it has to be operated at a higher efficiency than the Ni–Cd or the Ni–MeH batteries. This is mainly due to the low investment costs of the lead-acid cell. As a consequence, it has the lowest required energy benefit among the batteries in the table. Of course, this can change for mobile (i.e. transport) applications where the mass plays an important role.

In real life, a number of further important criteria would play decisive roles in storage-device selection: self-discharge rates, climatic conditions, safety and environmental concerns, and even commercial boundary conditions like established supplier relationships, to mention a few. Although this level of detail is beyond the scope of the present paper, note that most of these criteria can, in one way or the other, be taken into account in the NPV analysis. Self-discharge (leakage) does enter the Ragone-plot [2], or it can be taken into account in an effective charging efficiency. Climatic conditions might affect the lifetime. Safety and environmental concerns may come into play twice, in the expression for the benefit via a reduced demand and in the expression for the total lifecycle costs via disposal costs at the end of life. In any case, an economic comparison of different ESD types requires the complete NPV, including also constant terms that could be disregarded for an optimization of the single ESD.

The numbers for the supercapacitor should not be compared directly with the batteries, since the discharge time for supercaps is much shorter, and thus, the two different types of ESD must be associated with completely different applications. The assumption of identical utilization time $\alpha = 2000$ h per year means that the supercapacitor having the higher power capability is charged and discharged with much higher frequency. It therefore needs a lower minimal energy benefit. Note that the parameters used in Fig. 4 correspond to the supercapacitor and the lead-acid battery of Table 1.

Table 1
Comparison of optimization results for different ESD

	Lead-acid	Ni–Cd	Ni–MeH	Supercapacitor
$P_{\max}$ (W/kg)	100	200	200	3000
c_N (\$/kg)	5	30	30	50
τ (years)	2.5	4	5	2
η_c	0.85	0.8	0.85	0.95
K	11	5.8	6.5	24
$P = pP_{\max}$	45.2	112	108	1022
$b_e^{\min}$ (\$/kWh)	0.16	0.19	0.17	0.13

7. ESD with time-dependent power demand

If the power demand is time-dependent, $P_{\text{req}}(t)$, Eq. (14) has to be solved numerically for most realistic cases. We consider a specific example that is simple enough for an analytical treatment. Assume that the power request (per year) is equal to a constant P_1 during the time α_1, and equal to a constant P_2 for the duration α_2. The utilization time is now $\alpha = \alpha_1 + \alpha_2$. Without restriction we take $P_2 > P_1$. Eq. (14), which determines the optimum size N of the ESD becomes

$$K^{-1} = -\sum_{k=1}^{2} a_k y_k^2 \frac{e'(y_k)}{e^2(y_k)}, \quad (25)$$

where K is defined in Eq. (16), $y_k = N_k/N$ with $N_k = P_k/P_{\max}$, and $a_k = \alpha_k/\alpha$, with $k = 1$ and 2. It is clear that $N \geq N_2 > N_1$. The optimum system size N as a function of K can be obtained by evaluating the right-hand side of Eq. (25) as a function of N and plotting N versus K. In Fig. 5. We plotted the system size N for the battery as a function of K for various values of a_1 $(= 1 - a_2)$, and for $N_2 = 100$, $N_1 = 10$. Cases (a) and (e) correspond to cases with constant demands, $P_2 = 100$ and $P_1 = 10$, respectively. There is a cross-over between (a) and (e) for $N > 100$ at high K. However, for low K values (high investment costs) the optimum system size is limited by the minimum size that can supply the largest power request.

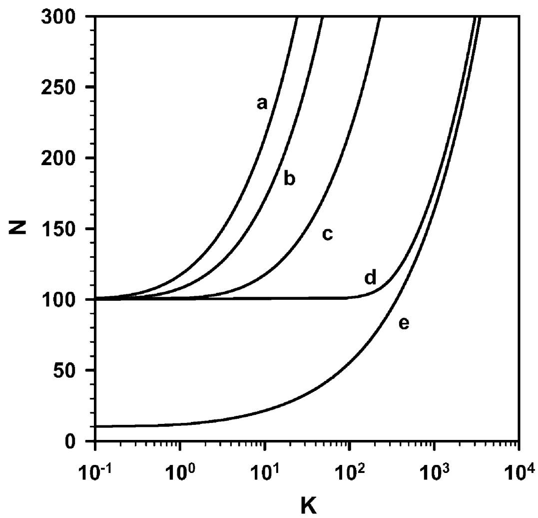

Fig. 5. Optimum system size as a function of K for an ESD that has to deliver two different power values, $P_1 = 10P_{max}$ and $P_2 = 100P_{max}$. The different curves correspond to different ratios of utilization times $\alpha_1 (= \alpha_1/(\alpha_1 + \alpha_2))$. (a) $\alpha_1 = 0$, (b) $\alpha_1 = 0.5$, (c) $\alpha_1 = 0.9$, (d) $\alpha_1 = 0.999$, (e) $\alpha_1 = 1$. For small K, the size is always limited by the maximum power request ($N \geq 100$ except for (e) where $N \geq 10$). Besides this, there is a clear cross-over from (a) to (e).

8. ESD for transportation

If an ESD is used for transportation purposes, one has to take into account the part of the power that is consumed for carrying the ESD itself. This implies that an increase of the ESD size (weight) leads not only to an increase in energy storage capability but also to an increase of the total power demand. For example, the results shown in Table 1 were established for stationary applications; it is, however, well-known that lead-acid batteries are an inferior technology for electric vehicles due to their heavy weight. We denote by P^0_{req} the power demand of the vehicle without the ESD's weight, and write the total power demand in the form:

$$P_{req} = P^0_{req}(1 + \kappa N), \tag{26}$$

where we assume a linear dependence of the additional power on the size of the ESD. The quantity κP^0_{req} is the additional power needed for transportation of one unit of the ESD. For simplicity, we consider again a constant vehicle power demand P^0_{req} during the utilization time α. The relation $N = P_{req}/P$ implies that

$$N = \frac{P^0_{req}}{P - \kappa P^0_{req}}. \tag{27}$$

In the dimensionless units ($p = P/P_{max}$, etc.), the optimization condition analogous to Eq. (18) becomes

$$\frac{1}{p - q}\left(1 + \frac{Kp}{e(p)}\right) = \text{min!} \tag{28}$$

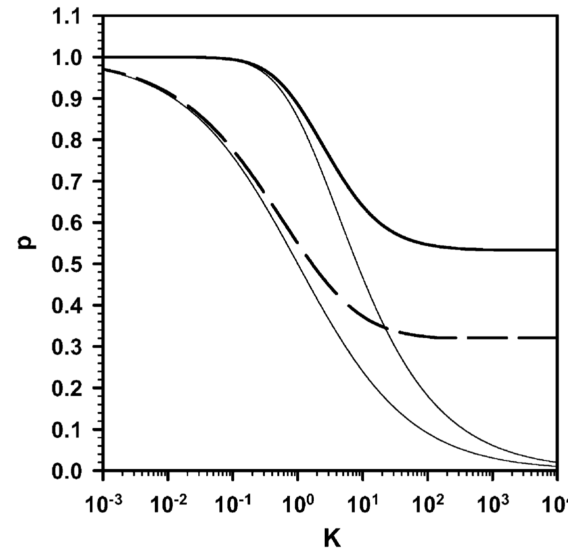

Fig. 6. Optimum operation point p for a mobile application (solid curve: battery; dashed curve: capacitor). The ratio of power needed for transportation of the ESD unit and the maximum power of a unit is assumed to be $q = 0.1$. The thin curves correspond to the case $q = 0$ for comparison.

with

$$q = \frac{\kappa P^0_{req}}{P_{max}}, \tag{29}$$

which is the ratio of the additional power needed for the transportation of the ESD unit and the maximum power of the unit. Clearly, it must hold $0 \leq q < 1$ and $p > q$, because the ESD must be able to supply the power for carrying its own mass. Solving Eq. (28) leads to

$$K = \frac{e^2(p)}{(q - p)pe'(p) - qe(p)}. \tag{30}$$

While this function can probably not be easily inverted, a plot of p versus K is shown in Fig. 6, for the case $q = 0.1$. The thin curves belong to $q = 0$ and are identical to those in Fig. 2. As one expects, the solutions for mobile and stationary applications differ only if the energy costs matter ($K \to \infty$). For strongly dominating investment costs ($K \to 0$), the power values converge towards $p \to 1$, i.e. maximum power capability.

The optimum power p for dominating energy costs ($K \to \infty$) is obtained with a brief calculation from Eq. (30). Vanishing of the denominator requires $p = 2\sqrt{q} - q$ for the battery, and $p = \sqrt{q}$ for the capacitor model. Using $N = q/\kappa(p - q)$ from Eq. (27) and (29) implies that

$$N = \frac{b}{\kappa}\frac{\sqrt{q}}{(1 - \sqrt{q})} = \frac{b}{\sqrt{q}(1 - \sqrt{q})}\frac{P^0_{req}}{P_{max}}, \tag{31}$$

with $b = 1/2$ for the battery, and $b = 1$ for the capacitor. The limit behavior of N as a function of q for dominating energy costs can easily be understood; $q = 0$ is identical to the case of a stationary application and shows the same behaviour

($N \to \infty$ for $K \to \infty$). On the other hand, $q \to 1$ means that the power remaining for transportation of the bare vehicle (without ESD) goes to zero, leading again to $N \to \infty$.

We mention that there is a variety of further interesting ESD optimization problems in transportation. For example, instead of modelling the payback per energy unit, the sales price that can be obtained by a customer could be a function of the maximum vehicle range and of available acceleration. Then, also b_e would be a function of p and can be included in the optimization in a straightforward way. This would, however, go beyond the scope of this paper.

9. ESD with an operation dependent lifetime

Up to this point, the lifetime used in our considerations was a fixed parameter determined by other components in the system than the ESD. In this section, we consider the case where the lifetime refers to the time-to-breakdown of the ESD itself. Now, τ and, consequently, d and K depend on the operation power p.

Considering a constant power P_{req} during the utilization time α, a maximization of the NPV (15) yields

$$Kp^2e' + \left(1 + p^2\left(\mu - \frac{1}{e}\right)K'\right)e^2 = 0 \tag{32}$$

with $K' = dK/dp$. We assume a lifetime model that leads to $K = K_0(1-p)^\gamma$, where the ESD breaks immediately (i.e. the lifetime vanishes) if the power reaches P_{max}; K_0 and γ are model parameters. We assume a constant discharge depth for all applications, such that the only reason of a lifetime decrease is due to an increased power. Reasonable ESD have γ-values with $0 \le \gamma \ll 1$. The NPV as a function of the operation point is plotted in Fig. 7, for $\gamma = 0.5$ and in units of $c_N P_{req}/P_{max}$. It is compared to the case with constant lifetime (dotted curve). The NPV is lower and the maximum is shifted to lower values of the operation power, as one expects because higher power now implies shorter lifetime in addition to lower efficiency.

Let us now compare maximization of the NPV with a maximization of the return on investment (ROI), defined as the ratio of the NPV to the present value of all costs, NPV/C_{LC}. The ROI is shown by the dashed–dotted curve in Fig. 7. A straightforward maximization yields:

$$Kp^2e' + \left(1 + \frac{pK'}{K}\right)e^2 = 0, \tag{33}$$

from which a relation between the optimum operation point p and K_0 is obtained. The optimum operation point as a function of K_0 is shown in Fig. 8, for NPV optimization (solid curve) and ROI optimization (dashed–dotted line). We assumed for illustration $\mu = 10$. Note that in general there exists a lower bound for the NPV or ROI in the sense that values below this bound are not acceptable, since the investment costs become too high. This bound is not necessarily at zero, since there might be other contributions to the NPV, which are independent of the energy. Nevertheless, we indicate by a dot in Fig. 8 the point where the NPV and the ROI vanish. From the definitions of NPV and ROI it follows directly that this point is at the same time the intersection point of theses curves. The result for constant lifetime, being equivalent to the result for minimization of lifecycle costs, is also indicated for illustration (dotted line). The operation point optimizing the NPV is shifted to lower values, as one expects. The ROI approach leads to results that differ only at low K_0 significantly from the approach of minimization of

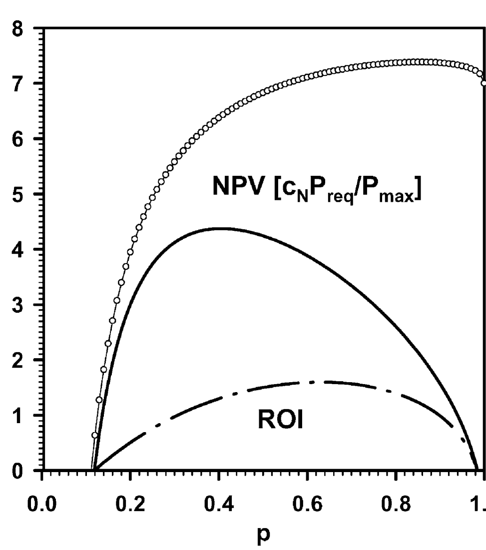

Fig. 7. Net present value as a function of p for the case where the lifetime depends on p. Parameter values: $K_0 = 1$, $\mu = 10$; solid curve: $\gamma = 0.5$; dotted curve: $\gamma = 0$ (constant lifetime); dashed–dotted curve: ROI (dimensionless units) for $\gamma = 0.5$.

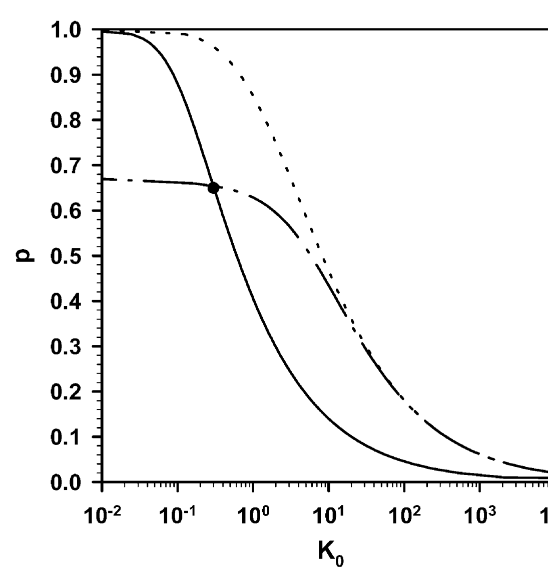

Fig. 8. Optimum operation point p as a function of K_0 with $\gamma = 0.5$, $\mu = 10$, and for the case where the lifetime depends on p (solid curve). Dashed–dotted curve: operation point from ROI maximation; the black dot indicates the zero of the NPV and ROI. Dotted curve: $\gamma = 0$ (constant lifetime).

lifecycle costs, i.e. at large power. This is understood by recognizing that in our case this approach can be interpreted as a minimization of average costs per effective lifetime, $C_{LC}/d =$ minimum. At small p this approximates well the constant lifetime case. At p close to 1, the decrease of d leads to an increase of C_{LC}/d. Consequently, the optimum operation point is significantly lower than for the constant lifetime case. The largest possible value of p for maximum ROI is given by $1/(1+\gamma)$. There is generally a large difference between the result of the NPV optimization and the ROI optimization. We conclude this section with the remark, that for $\mu < 2$ the example shown in the figure ($\gamma = 1/2$) can have a global non-quadratic maximum of the NPV which is at the boundary $p = 1$. Since this example is mainly for illustration we will not go further into details.

10. ESD with an operation dependent benefit

Until now the benefit B_{LC} has been associated with the monetary value of the energy supplied by the ESD. Benefits can, however, be given in terms of other values that characterize the utility derived by using the ESD. In this section, we model the benefit by the price a customer is willing to pay for the ESD. This price depends on the operational performance.

For illustration, consider an ESD used as an uninterruptible power supply (UPS). The UPS is obviously more useful the longer its discharge time (for constant P_{req}). As short power interruptions are much more frequent than long ones, the price of the ESD increases sublinearly with the discharge time, t. We model the dependence of the benefit on the discharge time by $B_{LC}(t) = B_0 P_{req}\sqrt{t}$ with a constant B_0. The energy costs are negligible for an UPS such that the restriction to $K = 0$ is appropriate. Then, the NPV becomes

$$\mathrm{NPV} = \frac{c_N P_{req}}{P_{max}}\left(\delta\sqrt{\frac{e}{p}} - \frac{1}{p}\right), \tag{34}$$

where $\delta = B_0 P_{max}\sqrt{(E_0/P_{max})}/c_N$ is the ratio between the customer benefit per unit for the characteristic discharge time E_0/P_{max}, and the investment costs per unit. It holds $\delta = B_0\sqrt{(Q_0 U_0^3/4R)}/c_N$ for the battery and $\delta = B_0\sqrt{(C/4R)}\,U_0^2/c_N$ for the capacitor (cf. Section 3). The NPV is easily optimized leading to a relation between optimum operation point and the parameter δ:

$$\delta = \frac{2\sqrt{e/p}}{e - pe'}. \tag{35}$$

The relation (35) is plotted in Fig. 9, for both battery and capacitor. For the capacitor, the simple expression $p = 4/(4+\delta^2)$ is obtained. Obviously, for the same δ the capacitor is optimal at lower p than the battery due to the lower energy efficiency which leads to a shorter discharge time. In the figure, the points where the NPV vanish are indicated by dots. The values of (p, δ) corresponding to zero NPV are $(0.889, 1.3)$ and $(0.5, 2)$ for the battery and the capacitor, respectively. The absolute value of the NPV will, in general, contain constant contributions which are not included in Eq. (34).

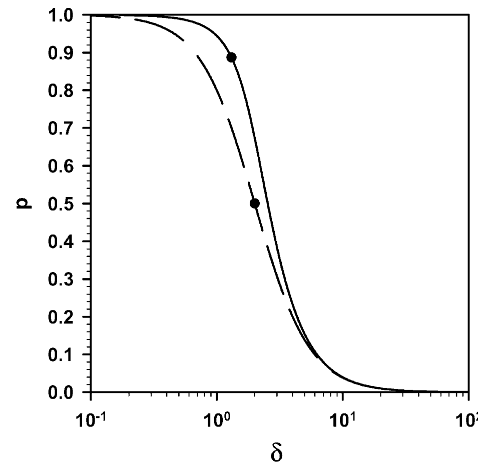

Fig. 9. Optimum operation point p as a function of δ; battery (solid curve) and a capacitor (dashed curve). The dots indicate the vanishing of the NPV, which becomes nageative at low δ.

We conclude this section by providing a numerical example. Consider an ESD with $E_0 = 30$ Wh/kg, $P_{max} = 100$ W/kg, $c_N = 5$ \$/kg. A typical value for B_0 is $250\,\$/\mathrm{kW}\sqrt{\mathrm{h}}$. One obtains then $\delta = 2.7$ with a corresponding optimum operation point $p = 0.45$. In other words, the optimum mass of the battery is 22 kg.

11. Conclusion

We have shown that the energy–power relation of an ESD (Ragone-plot) can be used for the calculation of the optimum operation point, or the optimum size of an ESD for a given application. A number of illustrative examples have been discussed. In the most simple case, the only economical parameter appearing is the ratio between the energy costs and investment costs. This parameter determines the balance between maximum efficiency and maximum power. The optimization depends on the specific application. We illustrated this for applications with dynamic load and for mobile applications. For fixed lifetime and for a benefit given in terms of the energy supplied by the ESD, the optimization task is equivalent to a life cycle cost minimization. On the other hand, a lifetime which depends significantly on the operation point requires knowledge on the benefits in order to optimize the NPV. It has been shown, that the optimization of the ROI, which does not require knowledge on the revenues, may lead to results that differ significantly from the NPV optimum. For optimization of a commercial product, the NPV optimization should be preferred from a rational point of view. Finally, we discussed a simple case of an UPS where the benefit depends on the operation point.

It should be clear that the present approach is rather general, and our examples can be generalized in a straightforward way.

For example, the NPV of an ESD for electric vehicles can now easily be written down, with a benefit depending on the range (discharge time) and the accelleration (related to the power), and including operation dependent lifetime as well as energy costs. One can also consider several variables leading to a multi-dimensional optimization problem, and one can include constraints. This is from a conceptional point of view straightforward, and leads to a realistic optimum design. In many cases, our approach can easily be extended to other types of power supplies. For example, fuel cells are treated similar to batteries with taking for η the total system efficiency as a function of power density.

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